Technical Note

On the second-order slow drift force spectrum

J. A. P. Aranha
Department of Naval and Ocean Engineering, EPUSP, São Paulo, Brazil

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A. C. Fernandes
Petrobrás, Rio de Janeiro, Brazil

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In this paper, it is shown that the slow drift force spectrum of a floating body, obtained from the exact quadratic transfer function, is flat in the low-frequency range of interest and can be written in the form \( S_\tau(\mu) = S_\tau(0) + O(\mu^2) \), where \( S_\tau(0) \) can be computed from the known drift force coefficient in harmonic waves and the wave energy spectrum. It is also shown here that a special and normally used form of Newman’s approximation for the exact quadratic transfer function has an error of the form \([1 + O(\mu^2)]\) at low frequencies. Copyright © 1996 Elsevier Science Ltd.

1 INTRODUCTION

Low-frequency wave excitation on a floating body can be described by the so-called quadratic transfer function \( T(\Omega_1; \Omega_2) \), namely the force that appears at the ‘difference frequency’ \( \Delta \Omega = \Omega_2 - \Omega_1 \), in the second-order interaction between two harmonic waves with unit amplitude and frequencies \( \Omega_1 \) and \( \Omega_2 \), respectively.

The numerical computation of \( T(\Omega_1; \Omega_2) \) is difficult, since one needs to evaluate not only the quadratic interaction of the linear potential, but also to compute the second-order potential at the difference frequency \( \Delta \Omega \). Observing that practical interest is focused on small values of \( \Delta \Omega \), Newman\(^1\) proposed the approximation

\[ T(\Omega_1, \Omega_2) \approx T(\Omega_1 + \alpha \Delta \Omega, \Omega_1 + \alpha \Delta \Omega) \equiv D(\Omega_1 + \alpha \Delta \Omega) \]

with \( 0 \leq \alpha \leq 1 \) and \( D(\ldots) \) being the drift force coefficient in harmonic waves.

If \( T(\Omega_1; \Omega_2) \) is known, one can compute the low-frequency force spectrum \( S_\tau(\Delta \Omega) \) and so the pertinent parameters of the response. In particular, if the low-frequency damping is small in the horizontal \( x \)-motion, its RMS value is given by the expression

\[ \sigma_x = \sqrt[4]{\frac{\pi}{4 \zeta} \frac{S_\tau(\Omega_n) \Omega_n}{R^2}} \]  \hspace{1cm} (1a)

where \( \Omega_n \) is the small natural frequency, \( \zeta \) is the percentage of the critical damping and \( R \) the restoring coefficient. In the context of this paper, the important point in the above expression is to make clear that the function \( S_\tau(\ldots) \) needs to be computed only for small values of its argument.

The purpose of this paper is to show that \( dS_\tau/d\Omega \) is zero at \( \Omega = 0 \) and so \( S_\tau(\Omega) \) is ‘flat’ in the region of interest. If \( \Omega_0 \) is the typical wave frequency, for example the sea spectrum peak frequency, one can introduce the variables:

\[ \omega = \Omega / \Omega_0 \]
\[ \mu_n = \Omega_n / \Omega_0 \]
\[ \mu = \Delta \Omega / \Omega_0 \]  \hspace{1cm} (1b)